GNSS NLOS Pseudorange Correction based on Skymask for Smartphone Applications

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ABSTRACT

In urban canyons, smartphone users experienced great inconvenience in inaccurate GNSS positioning. The development of 3D mapping aided (3DMA) GNSS is regarded as one solution for this challenge. Several ideas in 3DMA GNSS are proposed including, 1) non-light-of-sight (NLOS) measurement exclusion based on the 3D building model, 2) GNSS shadow matching, 3) ray-tracing based 3DMA GNSS, 4) likelihood-based 3DMA GNSS, etc. Recently, researches show the integration of these 3DMA GNSS methods can enhance the positioning accuracy. For example, GNSS shadow matching integrates with likelihood-based approach can reach a performance of less than 10 meters in most urban areas of cities. However, the performance of this state-of-the-art method is not as satisfactory in the deep urban canyons of mega Asian cities such as Hong Kong. In deep urban canyons, the number of NLOS affected measurements became excessive. About 75% of total measurement could be affected by NLOS. To achieve 10 meters level of positioning performance, the

NLOS measurement must be corrected. Ray-tracing based 3DMA GNSS aims to correct the pseudorange delay caused by NLOS reflection. It is proven effective in the urban canyons in Tokyo, Japan. However, its computation load is immense. We proposed a skymask (which is the skyplot with building boundaries) aided NLOS correction method, which can be regarded as an accelerated ray-tracing 3DMA GNSS method. Instead of using this skymask method standalone, this paper integrates the state-of-the-art 3DMA GNSS with our proposed skymask aided method. According to the experiment results, the NLOS correction generated by the skymask method further improves the performance of the integration of GNSS shadow matching and likelihood based method.

INTRODUCTION

People relies heavily on the GNSS-enable application to navigate himself or herself to go to the destinations. However, GNSS positioning is greatly challenged by the notorious multipath effect and non-line-of-sight (NLOS) reception [1]. These effects are caused due to the signal blockage and reflection by and on the buildings. In the other words, the more urbanized the city is, the more challenge on the GNSS positioning is. This is one of the current headaches of the smartphone service providers such as Apple and Google. As a result, the solutions for multipath and NLOS are urgently needed.

Other than the conventional approaches, one of the innovative solutions is to making use of 3D city models. Since the rise of smart cities, the 3D city models become widely available, especially for highly urbanized cities including Hong Kong, New York, Tokyo and London. These models can be used to effectively simulate the GNSS signal transmission in the urban areas. The methods that used the 3D mapping database to facilitate the GNSS positioning are called 3D mapping aided (3DMA) GNSS [2]. Dr Paul Groves from UCL proposes to depict the building boundaries on the GNSS skyplot [3]. In the other words, the skyplot have an elevation mask angle in each azimuth angle. This skyplot with building boundaries, which we called Skymask, can be used to classify the visibility (as LOS/NLOS) of GNSS satellite. Thus, he proposed one of the most of well-known 3DMA GNSS, the GNSS shadow matching (SDM) [3]. By matching the satellite visibilities, the algorithm can effectively determine which side of the street that the receiver is located. However, in the deep urban canyon with several paralleling narrow streets, the SDM could fall into a faulty estimation due to numerous of local minima [4]. Thus, a likelihood based 3DMA GNSS is proposed to incorporate the pseudorange measurement in a hypothesis based positioning method [5]. It is interesting and important to note the SDM and likelihood are complementary while the former and latter performs well in across- and along-streets, respectively. Thus, their integration is proposed and regarded as the state-of-the-art 3DMA GNSS method at this moment [6]. However, it cannot always perform satisfactory in the deep urban canyons in Asian mega cities, such as Hong Kong. The challenge is the excessive NLOS affected measurements [7]. Figure below gives an example a typical urban canyon in Hong Kong. As can be seen, 20 out of 27 pseudorange measurements are affected by NLOS reflection. In the other words, nearly 75% of the measurements are largely biased. Thus, instead of excluding or de-weighting the NLOS measurement, we believe it is should be corrected.

Figure 1: Example of LOS/NLOS measurements (GPS/GLONASS/Galileo/Beidou) collected by a smartphone in a typical urban canyon in Hong Kong. (left) Skymask. The red and green circle indicates NLOS and LOS measurements, respectively. (middle) Bird-view of the data collection site. (right) Street view of the site.

One idea is to correct the NLOS affected pseudorange by signal ray-tracing (RT) simulation based on 3D building model [8]. This ray-tracing based 3DMA GNSS method can obtain accurate positioning solution even in the environment of deep urban canyon. However, it comes along with computation load, which is very difficult to be used for smartphone users [9]. In addition, it also requires a calibration if different format of 3D building model is used. These two drawbacks limit the potential of the RT-3DMA GNSS. To uniform the format of different 3D building models, we learn from SDM to employ the Skymask as the standard format to restore 3D building model. The reason is its tiny-memory size, which makes it favorable for smartphone application. To reduce the computation load of RT simulation, we propose a new algorithm to detect the NLOS reflect point from the Skymask. The proposed Skymask not only contains the elevation mask angle but also contains its building height in each azimuth angle. Based on our previous finding in [10], the NLOS affected pseudorange can be corrected if we can obtain the building height (that reflected the GNSS signal transmission), elevation, and azimuth angle of the measurements. As a result, the RT simulation can be replaced by the proposed Skymask based NLOS correction. The previous result [10] was tested using u-blox M8T which provides relatively stable pseudoranges comparing to that of smartphone chip-level. This paper uses Samsung Galaxy Note8 (Qualcomm Snapdragon 835) with single-frequency GPS, GLONASS, GALILEO, and BeiDou constellation capabilities and the Xiaomi Mi8 (Broadcom BCM47755 chip) with dual-frequency GPS, GLONASS, GALILEO, BeiDou, and QZSS constellation capabilities are employed to record the raw pseudorange measurements in RINEX format and the NMEA data provided by the smartphone directly. The RINEX data is then post-processing and evaluate the performance of various 3DMA GNSS methods. Instead of using this proposed skymask method standalone, this paper integrates the state-of-the-art 3DMA GNSS [11] with our proposed skymask aided method. According to our experiment results, the proposed Skymask 3DMA algorithm can achieve positioning accuracy with about 10 meters error, which the positioning accuracy is competitive to RT-3DMA GNSS. However, the computation load diminishes significantly, where the computation duration for the proposed Skymask 3DMA is about one-third of that of RT-3DMA GNSS.

The contributions of this paper are summarized as following:

1. This paper proposes to use skymask as a standard correction format to detect and correct NLOS affected pseudorange measurements for smartphone applications. Since Skymask has already in-used due to the implementation of GNSS SDM, it does not increase additional cost for SDM user.
2. This is the first work proposes to correct NLOS affected pseudorange using skymask and NLOS delay model, which is a low computational load algorithm comparing to ray-tracing simulation.

3. The proposed skymask aided method can integrate with the state-of-the-art 3DMA GNSS to enhance positioning accuracy.

OVERVIEW OF THE PROPOSED ALGORITHM

The skymask 3DMA GNSS is used with a hypothesized candidate positioning method. It uses the measured and simulated pseudorange are used to describe the likelihood of the candidate. At candidates where the signal that predicted to be NLOS from the skymask. The error that cause by reflection from the building will be corrected based on the enhanced skymask format to provide correction on range level. Figure 2 shows the use of skymask NLOS correction with the state-of-the-art 3DMA GNSS [12]. This paper will not describe the state-of-the-art 3DMA GNSS method but to focus on how to integrate the proposed NLOS correction in it.

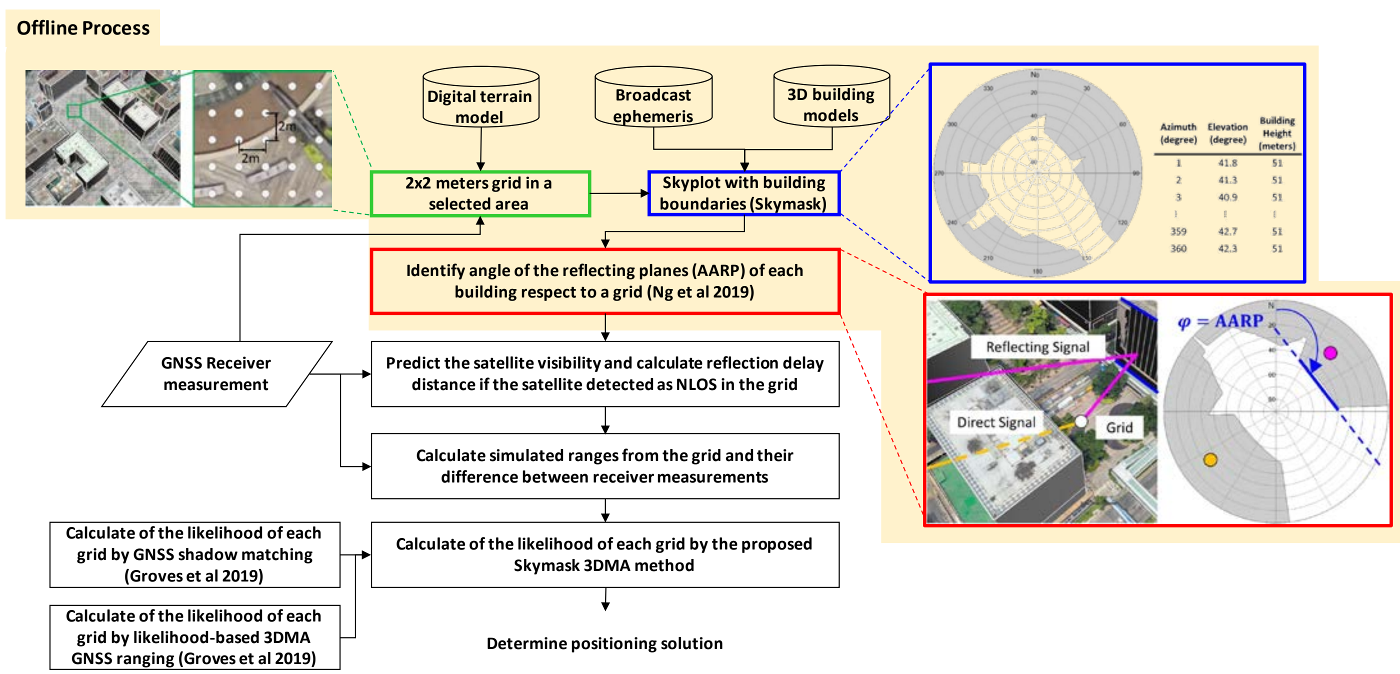


Figure 2: Flowchart of integrating the proposed Skymask 3DMA algorithm

A circular search area of center at the NMEA position solution with radius 40m is defined. Within the searching area, positioning candidate are distributed in grid with spacing of 2m. On each candidate, the azimuth angle of the reflecting planes (AARP) of the building models surrounded the candidate will be determined from corresponding skymask. The skymask of certain area can be pre-computed from the 3D building model on the server side. For a practical implementation, the smartphone can request the skymask of an area from server when positioning. The AARP is the parallel direction of the building plane on skymask, this value can also be pre-computed, in practical implementation. The detail of an algorithm to automatically determined AARP can be found at [13].Then, the satellite visibility is predicted using skymask. If it is detected as NLOS, the reflecting point can be detected using the AARP [10]. As a result, the horizontal distance from the grid to the reflection plane can be retrieved from the enhanced skymask which also provide the building height of corresponding azimuth. The actual position of the reflecting point can be obtained by the horizontal distance and elevation angle. The reflection delay distance, $\varepsilon^{j}_{refl(i)}$, is calculated by the sum of geometric distance between reflecting point and satellite and geometric distance between candidate and reflecting point then subtract the geometric distance between candidate and satellite. Therefore, the simulated pseudorange, $\hat{\rho}^{j}_{i}$, of the j-th satellite on i-th candidate can be found by geometric range between candidate and satellite, R^{j}_{i}. With other error terms like ionospheric delay [14], I^{j}, tropospheric delay [15], T^{j}, satellite constellation delay, $c\delta t^{j}$, where c is the speed of light, and reflection delay if it is a NLOS predicted signal, $\varepsilon^{j}_{refl(i)}$.

$$\hat{\rho}^{j}_{i} = R^{j}_{i} + c\delta t^{j} \ + I^{j} + T^{j} + \varepsilon^{j}_{refl(i)} \tag{1}$$

To eliminate the receiver delay, all the measurements will be single differenced (between the master and slave satellites) once which categorized by each constellation. The master satellite, $r(j)$, for each constellation is selected by the LOS satellite with highest elevation angle to minimize the error when difference across all other satellites. Therefore, the ranges difference, D^{j}_{i}, between the measurements, $\tilde{\rho}^{j}$, and the simulated ranges can be found.

$$D^{j}_{i} = \left|\left(\tilde{\rho}^{j} \ - \tilde{\rho}^{r(j)}\right) - \left(\hat{\rho}^{j}_{i} - \hat{\rho}^{r(j)}_{i}\right)\right| \tag{2}$$

Each candidate is scored with the likelihood, Λ_{i}, same as [5] proposed. Full details of the scoring with measurements error covariance matrix is described in [5].

The position solution is obtained from the weighted average of all candidates' likelihood.

$$x(t) = \frac{\sum_{i}(\Lambda_{i}(t)P_{i}(t))}{\sum_{i}\Lambda_{i}(t)} \tag{3}$$

In short, in this paper the positioning process is similar to the likelihood-based ranging 3DMA GNSS. The only difference is the NLOS measurement modelling, the skymask 3DMA is using the enhanced skymask with geometry to provide the correction. While the likelihood-based 3DMA using the statistical way to model the NLOS predicted signal.

On the result comparison on different positioning algorithms and their integrating results, the skymask 3DMA is integrated by shadow matching [12] and likelihood-based ranging method [5] with the hypothesis domain integration method proposed in [12].

EXPERIMENTS RESULTS AND ANALYSIS

Several sets of experimental data were collected in Hong Kong, the Samsung Galaxy Note 8 (Qualcomm Snapdragon 835) and the Xiaomi Mi 8 (Broadcom BCM47755 chip) were employed. The output rate of raw measurements is 1 Hz.

The collected raw data is then pro-processed to evaluate the performance of each method. The algorithms compared are listed as following:

1. WLS: weighted-least-squared [16]

2. SDM: GNSS shadow matching [12]
   In (4), $\Lambda_i(t) = \Lambda_{i,SDM}(t)$, where the likelihood by SDM is calculated based on [12].

3. LBR: likelihood-based 3DMA GNSS ranging [5]
   In (4), $\Lambda_i(t) = \Lambda_{i,LBR}(t)$, where the likelihood by likelihood-based ranging method is calculated based on [5].

4. SKY: the proposed skymask 3DMA GNSS [13]
   In (4), $\Lambda_i(t) = \Lambda_{i,SKY}(t)$.

5. SDM + LBR: hypothesis domain integration of shadow matching and likelihood-based 3DMA GNSS ranging
   In (4), $\Lambda_i(t) = \Lambda_{i,SDM}^{W_p}(t) \cdot \Lambda_{i,LBR}(t)$ based on [12]. The W_p is calculated by the score of SDM [12].

6. SDM + SKY: hypothesis domain integration of shadow matching and skymask 3DMA
   In (4), $\Lambda_i(t) = \Lambda_{i,SDM}^{W_p}(t) \cdot \Lambda_{i,SKY}(t)$.

7. SDM + LBR + SKY: hypothesis domain integration of shadow matching, likelihood-based 3DMA GNSS ranging, and skymask 3DMA
   In (4), $\Lambda_i(t) = \Lambda_{i,SDM}^{W_p}(t) \cdot \Lambda_{i,LBR}(t) \cdot \Lambda_{i,SKY}(t)$.

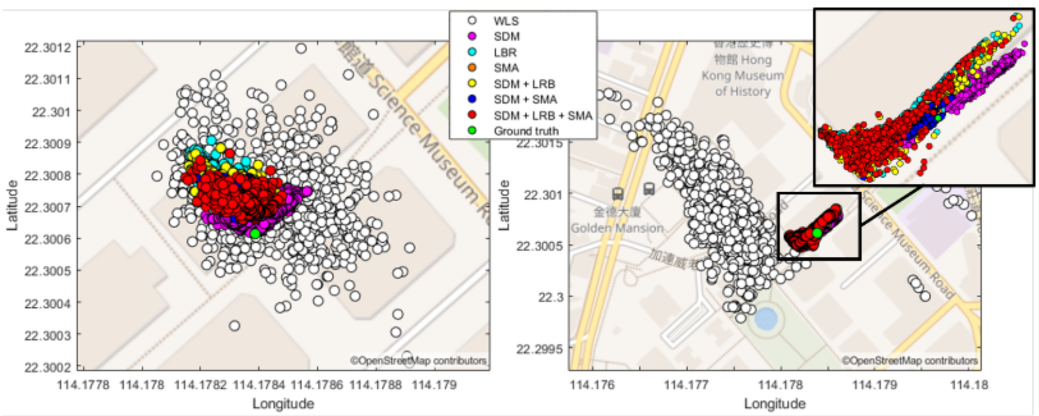
The experiment setups are shown on Figure 3 and Table 1. Noted that the building height to street width ratio is calculated by $building\ height/street\ width$, which means that the value is higher, the street is narrower with taller building surrounded and the environment is more challenging for positioning.

Figure 3: Location and pedestrian behavior in the experiments and smartphones used.

Table 1: Experiments setup

Experiment	District	Duration (seconds)	Building height to street width ratio ($\frac{building\ height}{street\ width}$)
1: static	Tsim Sha Tsui	916	3.88
2: static	Tsim Sha Tsui	687	2.17
3: dynamic	Tsim Sha Tsui	66	0.68
4: dynamic	Yau Ma Tei	101	2.83
5: static	Yau Ma Tei	605	2.81

The experiment 1 is a static experiment. Figure 4 and Table 2 shows the positioning results and RMS error. The environment of this experiment is challenging that the street is narrow with about 10m width while surrounded buildings are 40m height. In here, the results of stand-alone shadow matching and the integrated positioning on shadow matching, likelihood-based 3DMA ranging, and Skymask 3DMA are perform better. Also noted that, the position integration on shadow matching and Skymask 3DMA perform excellent on the Samsung Galaxy Note 8, the reason may due to the device-dependent on different algorithms.



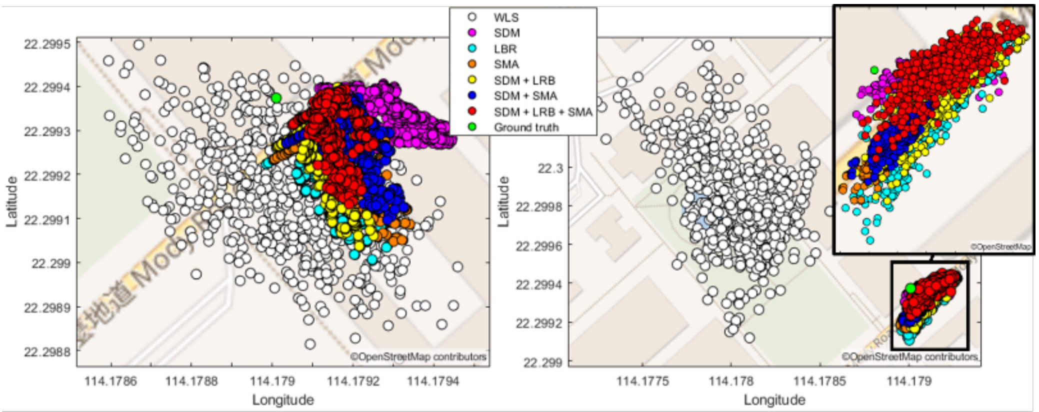
(a) Xiaomi Mi 8 (b) Samsung Galaxy Note 8

Figure 4: (a) positioning results of experiment 1 by Xiaomi Mi 8; (b) positioning results of experiment 1 by Samsung Galaxy Note 8

Table 2: RMS error of experiment 1 using different algorithms

Receiver	RMS error (m)	WLS	SDM	LBR	SKY	SDM + LBR	SDM + SKY	SDM + LBR + SKY
Xiaomi Mi 8	2D	26.61	11.54	18.51	17.25	15.53	14.22	12.79
	Along street	17.56	10.24	5.38	5.49	6.20	6.30	6.66
	Across street	20.00	5.32	17.71	16.35	14.24	12.74	10.92
Samsung Galaxy Note 8	2D	115.72	17.10	19.55	6.99	17.66	5.26	18.28
	Along street	67.28	17.01	17.44	6.11	16.21	4.76	16.64
	Across street	94.16	1.78	8.84	3.39	7.01	2.23	7.58

The experiment 2 is a static experiment. The experiment environment is a relatively open than that of experiment 1. Figure 5 and Table 3 show the positioning results and RMS error of experiment 2. From the positioning results, the all 3DMA integrated also perform well. Especially on Xiaomi Mi 8, the positioning results improve much, from about 24m by likelihood-based 3DMA or the Skymask 3DMA integrate with shadow matching, and improve to about 17m.



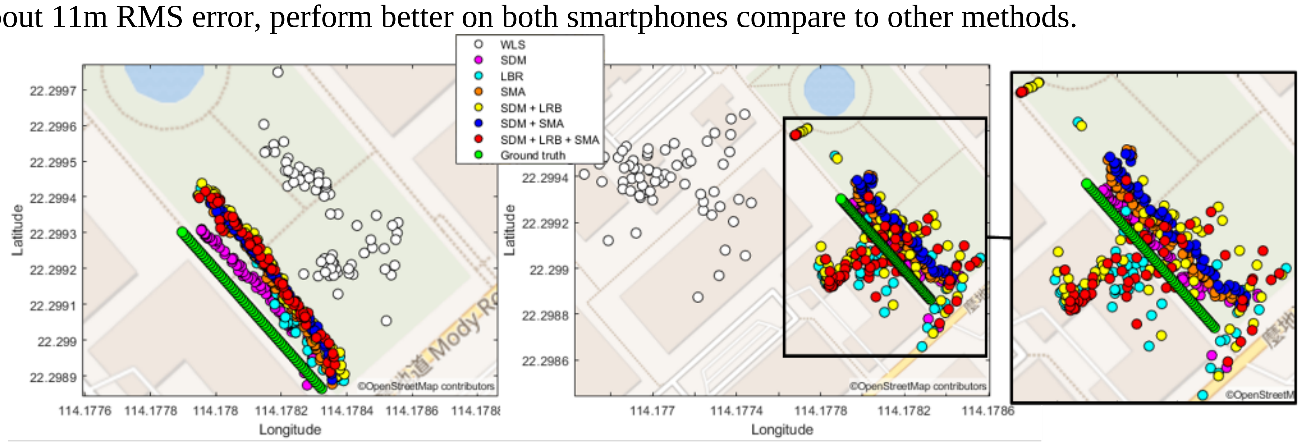
(a) Xiaomi Mi 8 (b) Samsung Galaxy Note 8

Figure 5: (a) positioning results of experiment 2 by Xiaomi Mi 8; (b) positioning results of experiment 2 by Samsung Galaxy Note 8

Table 3: RMS error of experiment 2 of different algorithm

Receiver	RMS error (m)	WLS	SDM	LBR	SKY	SDM + LBR	SDM + SKY	SDM + LBR + SKY
Xiaomi Mi 8	2D	32.15	31.48	23.96	24.94	21.05	23.67	17.35
	Along street	17.83	19.71	7.19	5.94	6.39	7.56	6.64
	Across street	26.75	24.54	22.85	24.22	20.06	22.43	16.03
Samsung Galaxy Note 8	2D	118.32	11.46	14.94	15.46	14.95	12.47	14.37
	Along street	42.88	8.00	8.90	9.06	10.13	5.67	11.37
	Across street	110.27	8.21	12.00	12.53	11.00	11.11	8.79

The experiment 3 is a dynamic experiment in a relatively open-sky environment. The walking distance of the experiment is 65m. Figure 6 and Table 4 show the positioning results and RMS error of experiment 3. In here, the stand-alone shadow matching with about 8m RMS error and integrated with Skymask 3DMA with about 11m RMS error, perform better on both smartphones compare to other methods.



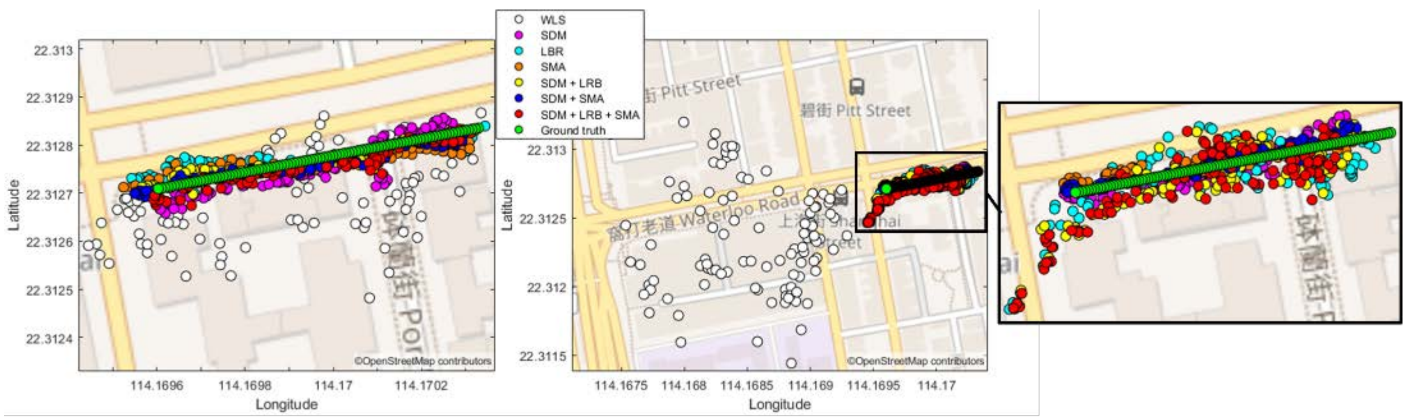
(a) Xiaomi Mi 8 (b) Samsung Galaxy Note 8 (c) Samsung Galaxy Note 8 (zoom-in)

Figure 6: (a) positioning results of experiment 3 by Xiaomi Mi 8; (b) positioning results of experiment 3 by Samsung Galaxy Note 8; (c) zoomed-in positioning results of experiment 3 by Samsung Galaxy Note 8

Table 4: RMS error of experiment 3 of different algorithm

Receiver	RMS error (m)	WLS	SDM	LBR	SKY	SDM + LBR	SDM + SKY	SDM + LBR + SKY
Xiaomi Mi 8	2D	38.67	8.18	12.55	10.45	14.47	11.79	13.70
	Along street	7.22	5.53	5.91	1.12	6.42	1.37	5.57
	Across street	37.99	6.03	11.08	10.39	12.97	11.71	12.51
Samsung Galaxy Note 8	2D	114.62	7.16	23.60	10.23	21.34	12.79	23.15
	Along street	98.43	4.02	12.87	3.44	11.47	4.04	12.65
	Across street	58.73	5.93	19.78	9.63	17.99	12.14	19.39

The experiment 4 is a dynamic experiment, positioning results and RMS error of two smartphones shows in Figure 7 and Table 5 respectively. The experiment take place in about 22m width of the street, with surrounding buildings height is about 45m. And the walking distance is about 76m. All 3DMA are perform well on this experiment, but it is observed that the shadow matching and Skymask 3DMA integrated perform better on both smartphones averagely with about 6m RMS error.



(a) Xiaomi Mi 8 (b) Samsung Galaxy Note 8 (c) Samsung Galaxy Note 8 (zoom-in)

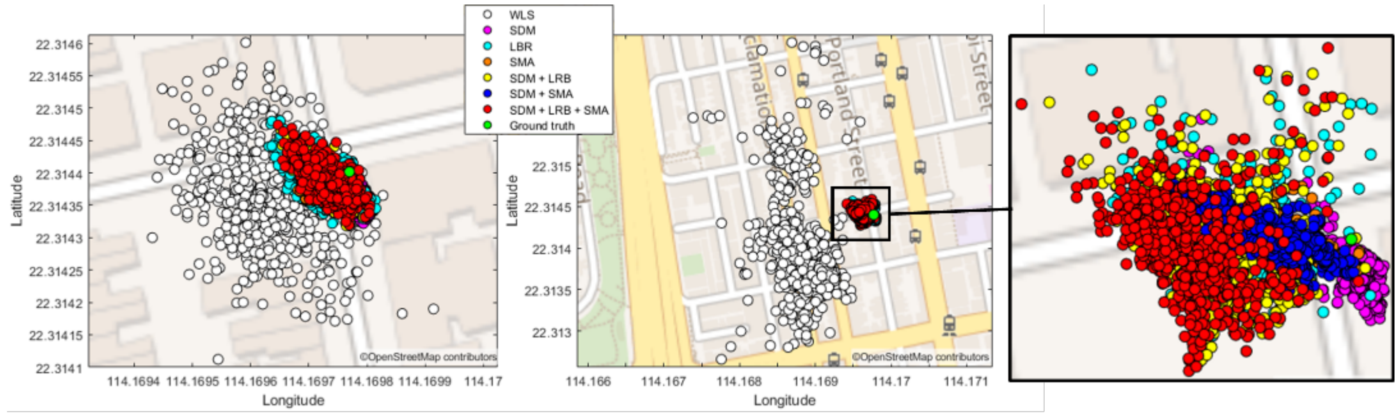
Figure 7: (a) positioning results of experiment 4 by Xiaomi Mi 8; (b) positioning results of experiment 4 by Samsung Galaxy Note 8; (c) zoomed-in positioning results of experiment 4 by Samsung Galaxy Note 8

Table 5: RMS error of experiment 4 of different algorithm

Receiver	RMS error (m)	WLS	SDM	LBR	SKY	SDM + LBR	SDM + SKY	SDM + LBR + SKY
Xiaomi Mi 8	2D	18.33	5.68	5.65	6.31	4.89	5.21	5.27
	Along street	14.57	4.51	5.01	5.75	4.67	4.93	4.90
	Across street	11.12	3.45	2.61	2.60	1.45	1.69	1.95
Samsung Galaxy Note 8	2D	165.52	7.91	9.96	5.20	10.60	5.97	12.13
	Along street	157.49	7.49	7.31	4.72	7.93	5.80	9.70
	Across street	50.91	2.55	6.76	2.20	7.03	1.43	7.28

The experiment 5 is a static experiment. The experiment take place in an extremely deep urban canyon which is also a common scenario in Hong Kong. The street width is about 10m, with surrounding buildings height is about 30m. And the experiment located in the intersection of two streets in perpendicular, which means there is no clear along or across street direction in the theoretical assumption of positioning algorithm. Figure 8 and

Table 6 shows positioning results and the RMS error of experiment 5 respectively. From the results, a similar conclusion of experiment 4, the shadow matching and Skymask 3DMA integrated results as well as integrated likelihood-based 3DMA perform slightly better than stand-alone shadow matching and likelihood-based 3DMA.



(a) Xiaomi Mi 8 (b) Samsung Galaxy Note 8 (c) Samsung Galaxy Note 8 (zoom-in)

Figure 8: (a) positioning results of experiment 5 by Xiaomi Mi 8; (b) positioning results of experiment 5 by Samsung Galaxy Note 8; (c) zoomed-in positioning results of experiment 5 by Samsung Galaxy Note 8

Table 6: RMS error of experiment 5 of different algorithm

Receiver	RMS error (m)	WLS	SDM	LBR	SKY	SDM + LBR	SDM + SKY	SDM + LBR + SKY
Xiaomi Mi 8	2D	18.77	6.13	7.19	7.38	5.73	5.87	5.66
	Along street	9.16	5.89	3.50	1.67	3.27	1.55	3.40
	Across street	16.38	1.70	6.28	7.19	4.70	5.67	4.52
Samsung Galaxy Note 8	2D	132.40	5.55	17.03	13.31	17.24	11.44	19.13
	Along street	73.70	4.23	6.10	5.28	6.04	4.35	6.15
	Across street	110.00	3.60	15.90	12.22	16.15	10.58	18.11

CONCLUSION AND FUTURE WORK

The proposed 3DMA GNSS using the enhanced skymask, to identify the reflecting point's angular position and pseudorange correction for the NLOS measurements. According to the experiments results with smartphone raw measurements, the positioning accuracy can achieve about 10m error. Also noted that the positioning accuracy of the state-of-the-art 3DMA GNSS can be further improved if the Skymask 3DMA is integrated with it. In the near future, the GNSS L5 measurement will be used in the proposed 3DMA GNSS method to mitigation the effect of multipath signals, which also a real challenge for urban positioning.

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